A new similarity measure to reveal individual differences and growth in implicit number conceptions

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Background

- How individuals think about numbers can be measured via psychological similarity ratings.[1,2,3]
- Previous multi-dimensional scaling (MDS) analyses reveal differing representations of number concepts due to maturation[2] and abacus expertise.[3]

We explore the use of similarity judgments to:
1) serve as a reliable snapshot of *individuals'* internal representations of number
2) eventually track changes in the relative salience of specific numerical properties for pre- and post-test use

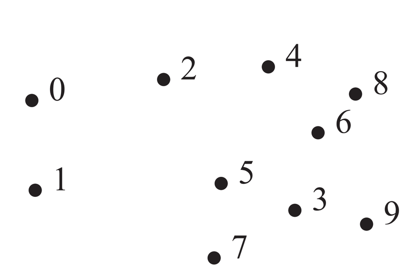
Research Questions

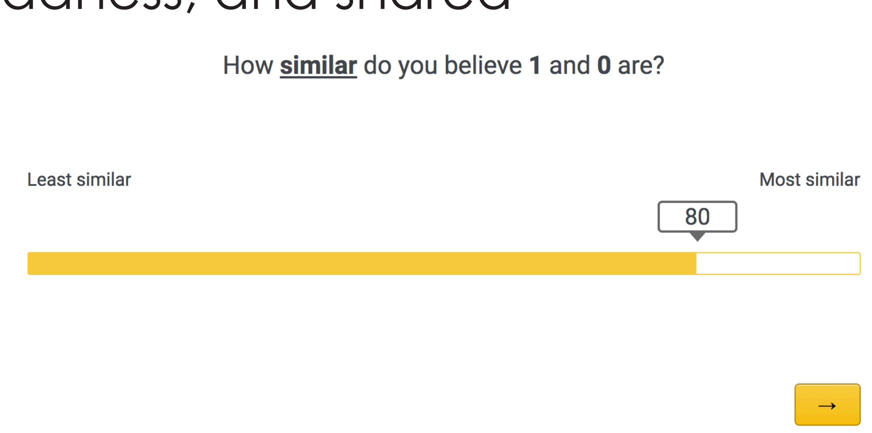
- **How are numbers organized in individuals' mental representations?**
- **Is the structure of individuals' number conceptions consistent across time?**
- **How do individuals' explicit knowledge of mathematical properties of numbers relate to their implicit representation?**

- In **Experiment 1**, we replicate earlier work using similarity judgments to capture the structure of numerical representations in a sample of adults on MTurk.
- In **Experiment 2**, we develop and test a new, expanded measure of individuals' conceptions of number.

Experiment 1: Replication

- 21 participants on Amazon's Mechanical Turk rated the similarity between all pairs of numbers from 0 to 9.
- This study replicated earlier work with adults: numbers were ordered from smallest to largest, but clustered into groups based on evenness, oddness, and shared factors.





Experiment 2: Extension

- We develop two matching 10-item sets of numbers to test the reliability of similarity judgments as an individual differences measure.

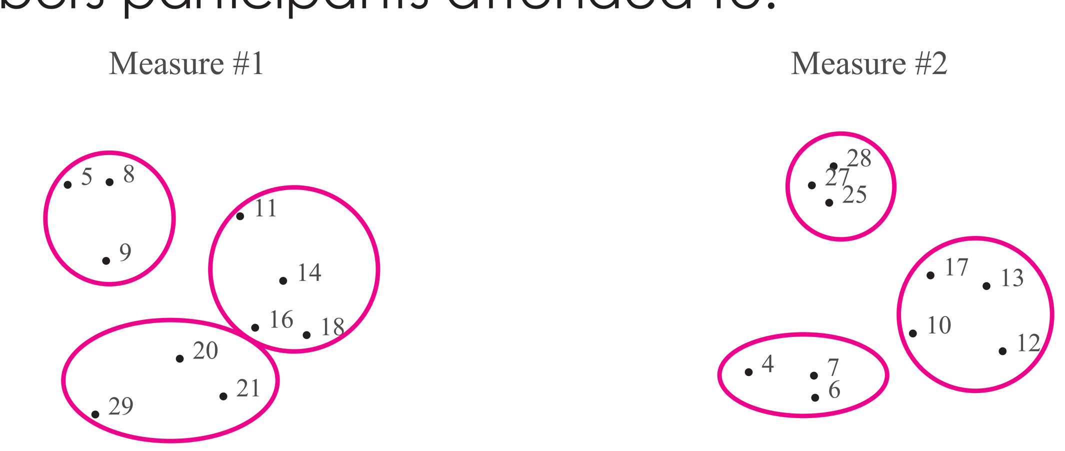
	MEASURE 1	MEASURE 2
Under 10 (3)	5 8 9	4 6 7
10-19 (4)	11 14 16 18	10 12 13 17
20-29 (3)	20 21 29	25 27 28
Even (5)	8 14 16 18 20	4 6 10 12 28
Odd (5)	5 9 11 21 29	7 13 17 25 27
Prime (3)	5 11 29	7 13 17
Multiple of 3 (3)	9 18 21	6 12 27
Multiple of 4 (3)	8 16 20	4 12 28
Multiple of 5 (2)	5 20	10 25
Multiple of 7 (2)	14 21	7 28
Perfect square (2)	9 16	4 25

84 adult participants completed:
1) all pairwise similarity judgments between the 20 items from both measures
2) an explicit math task categorizing all 20 numbers based on relevant mathematical properties
3) the Single Item Math Anxiety Rating Scale (SIMA)[4]

Results

- The majority of the relevant mathematical concepts were well understood and remembered by participants. However, the apparent salience of these concepts in making similarity judgments varied.
- The mean math anxiety score was 4.19 out of 10. Performance on the explicit math task was negatively related to math anxiety.
- MDS analyses revealed individual differences in the properties of numbers participants attended to:

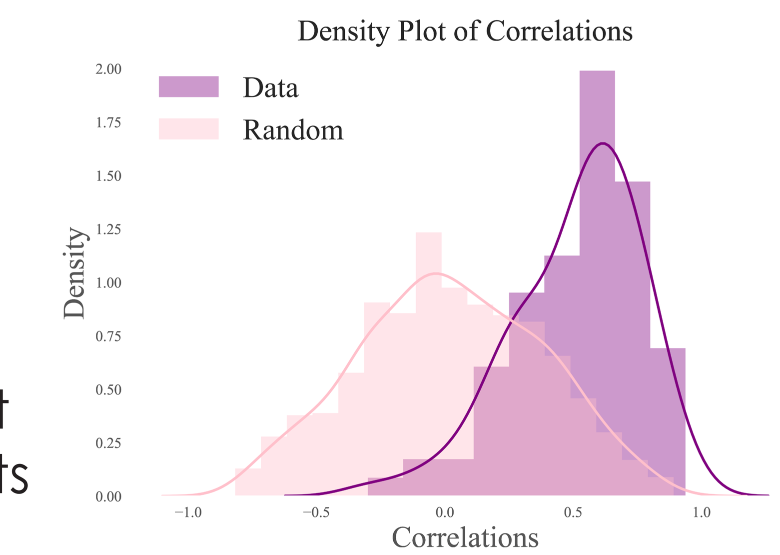
- attention to *magnitude*



- attention to *parity*

Results, cont.

- Weights assigned to each property were **highly correlated within individuals**, as compared to randomly generated similarity ratings:



- Performance on the explicit task was unrelated to weights from the implicit task.

Conclusions

- As seen in earlier studies, adults' conceptions of numbers include properties beyond mere proximity on the number line
- Individuals' explicit knowledge of numerical properties is not necessarily reflected in their implicit representations.
- Individuals differ in the relative salience of specific properties, and these differences appear consistent across measures.

Future Directions

- Measures from Exp. 2 may be used to track learning and intervention effectiveness.
- Given that similarity ratings are known to be highly consistent, yet context-dependent[5], priming individuals to think about specific properties of number may influence the way they represent numbers
- Future methods may employ pile-sorting to enable to inclusion of more items.
- How do conceptions of number *develop* and how do different curricula influence how math and numbers represented?

Acknowledgements

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References

1 Shepard, R. N., Kilpatric, D. W., & Cunningham J. P. (1975). *Cognitive Psychology*, 7, 82-138.
2 Miller, K. & Gelman, R. (1983). *Child Development*, 54(6), 1470-1479.
3 Miller, K. & Stigler, J. (1991). *Cognition and Instruction*, 8(1), 29-67.
4 Núñez-Peña, M. I., Guilera, G., & Suárez-Pellicioni, M. (2013). *Journal of Psychoeducational Assessment*, 20(10), 1-12.
5 Ross, B.H. & Murphy, G.L. (1999). *Cognitive Psychology* 38, 495–553.